

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Mechanics 5

MM05

Friday 23 June 2006 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Anne has made a simple pendulum which has a period of 4 seconds.

Show that the length of Anne's pendulum is $\frac{4g}{\pi^2}$. (3 marks)

- 2 A particle moves with simple harmonic motion on a straight line between the points P and Q . The amplitude of this motion is 0.3 m.

When the particle is 0.06 m from P , its speed is 0.9 m s^{-1} .

(a) Show that the period of motion is $\frac{2\pi}{5}$ seconds. (5 marks)

(b) Find the magnitude of the maximum acceleration of the particle. (2 marks)

- 3 A particle moves so that, at time t , its polar coordinates (r, θ) with respect to a fixed origin, O , are such that $r = \frac{a}{1 + 5 \cos \theta}$, where a is a constant.

(a) At the point A , the value of r is a minimum.

Show that at this point $r = \frac{a}{6}$. (2 marks)

(b) Show that $\dot{r} = \frac{5r^2}{a} \dot{\theta} \sin \theta$. (3 marks)

(c) Show that $\ddot{r} = \frac{5a\dot{\theta}^2}{36}$ when the particle is at A . (6 marks)

(d) The radial acceleration of the particle is $-\frac{\lambda}{r^2}$, where λ is a constant.

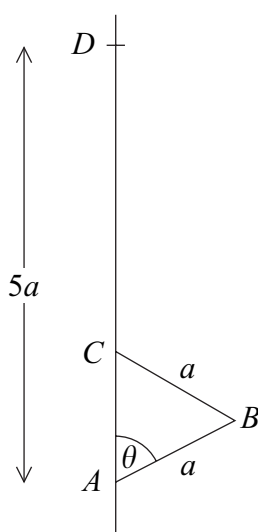
Find the speed of the particle at A in terms of λ and a . (5 marks)

- 4 A smooth rod passes through two points A and D , where D is a distance $5a$ vertically above A .

Two uniform rods, AB and BC , each of mass $2m$ and length a , are smoothly pivoted at B . The rod AB is smoothly pivoted at A , and the rod BC has the end C attached to a light smooth ring that can move freely on the vertical rod AD .

The ring, attached to the rod BC at C , is joined to the point D by a light spring, which has modulus of elasticity $6mg$, and natural length $3a$.

The rod AB is inclined at an angle θ to the vertical with the ring above A , as shown in the diagram.



- (a) Show that V , the total potential energy of the system, is given by

$$V = 4mga(1 - \cos \theta + \cos^2 \theta)$$

where the gravitational potential energy is taken to be zero at the level of A . (6 marks)

- (b) Find the two values of θ in the range $0 \leq \theta \leq \frac{\pi}{2}$ for which the system is in equilibrium. (4 marks)
- (c) Determine, for each of these values of θ , whether the system is in stable or unstable equilibrium. (4 marks)

Turn over for the next question

Turn over ►

- 5 A car travels along a straight horizontal road. At time t , its speed is v and the total resistance to motion has magnitude kv , where k is a constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant rate λ and at a constant speed V relative to the car.

The initial mass of the car and the fuel is M .

- (a) By considering linear momentum, show that the acceleration of the car along the road is

$$\frac{\lambda V - kv}{M - \lambda t} \quad (7 \text{ marks})$$

- (b) The initial speed of the car is zero. Find its speed at time t . (6 marks)
- (c) Initially, the mass of the fuel is 75% of the mass M . When $t = T_0$, all of the fuel has been burnt. Find T_0 in terms of M and λ . (3 marks)

- 6 A particle P , of mass $2m$, is suspended from a fixed point O by a light elastic string of natural length a and modulus of elasticity $8mn^2a$, where n is a positive constant. The particle is released from rest when $t = 0$ at a point A , where A is vertically below O and $OA = a$.

When the particle is moving with speed v , it experiences air resistance of magnitude $4mnv$.

- (a) The displacement of P below A at time t is x . Show that x satisfies the equation

$$\frac{d^2x}{dt^2} + 2n \frac{dx}{dt} + 4n^2x = g \quad (3 \text{ marks})$$

- (b) Find x in terms of n , g and t . (12 marks)
- (c) Hence show that the first time the particle comes to rest is when $t = \frac{\pi}{\sqrt{3}n}$. (4 marks)

END OF QUESTIONS